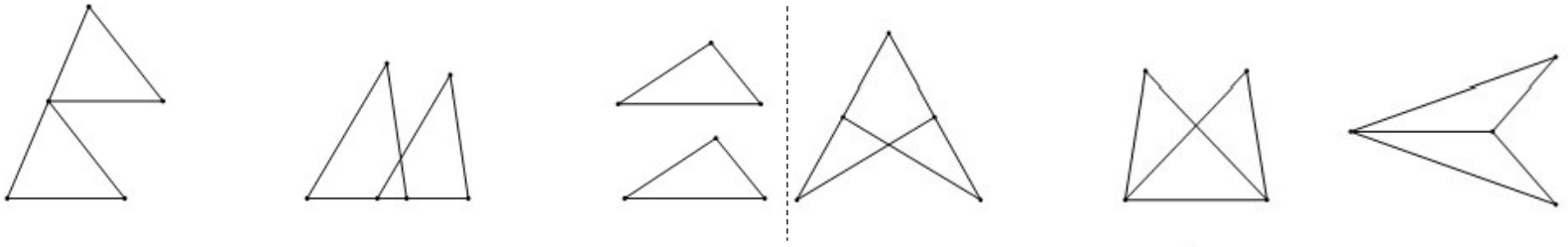
Mastery on Congruent Triangles

Prerequisites / Entry Requirements

- *Completion of the two "Congruent Triangles" lectures*
- *equivalent geometric experience*

Mastery Level Knowledge Starting Point / Core Points

Translation congruence type *Reflection (axisymmetric) congruence type*



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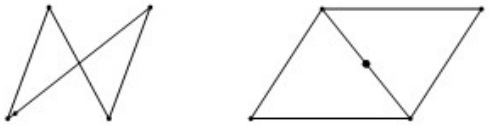
Rotation congruence type



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Composite types :more common in practice

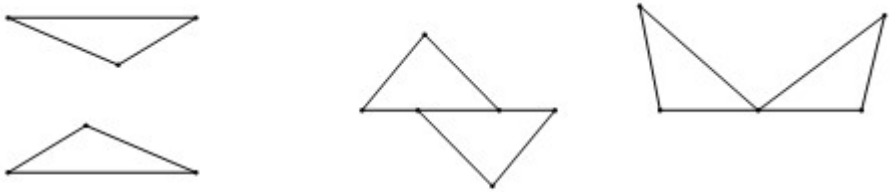


Fig. 4

Congruent Triangles Criterion Theorem

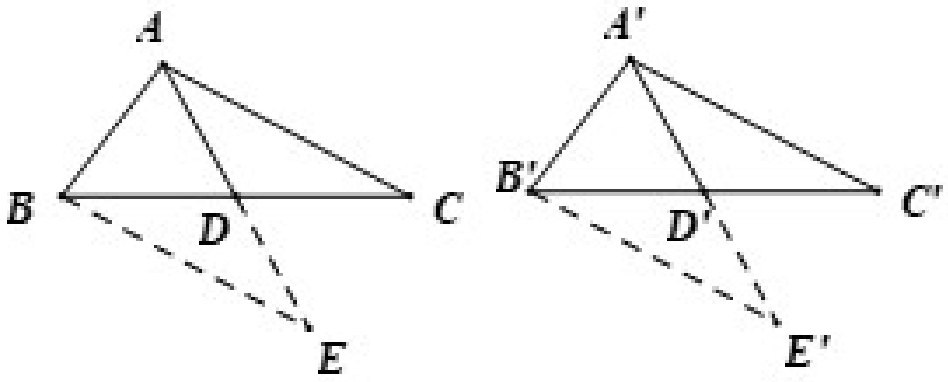
- *Side-Angle-Side (SAS)*
- *Angle-Side-Angle (ASA)*
- *Angle-Angle-Side (AAS) (corollary)*
- *Side-Side-Side (SSS): the rigidity (or stability) of a triangle*
- *Hypotenuse-Leg (HL): right triangles*

Standard Problem-Solving Pathway

detective-like, puzzle-solving mindset

Example 1 *Prove: If two triangles have two corresponding sides and the median to the third side equal respectively, then the two triangles are congruent.*

Given: In ⊿ABC and ⊿A'B'C', AD and A'D' are the medians to sides BC and B'C' respectively, with AB = A'B', AC = A'C', and AD = A'D'. Prove: ⊿ABC ≅ ⊿A'B'C'.



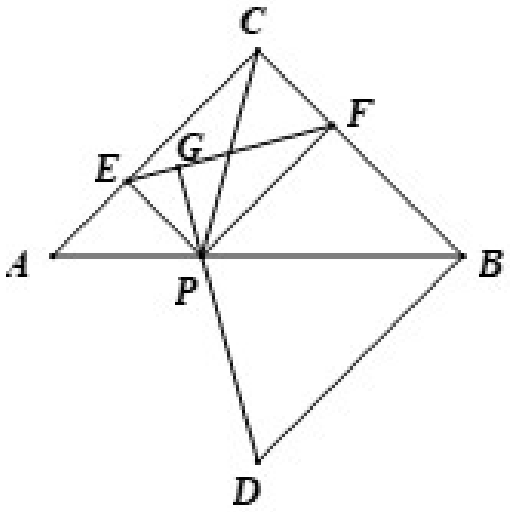
Key Idea / Approach

Apply the classic "double the median" technique

Proof:

Example 2 *Let P be an arbitrary point on the hypotenuse AB of the isosceles right triangle ACB. From P, drop perpendiculars: PE ⊥ AC at E, PF ⊥ BC at F. Then PG ⊥ EF at G. Extend GP beyond P and take point D on the extension such that PD = PC.*

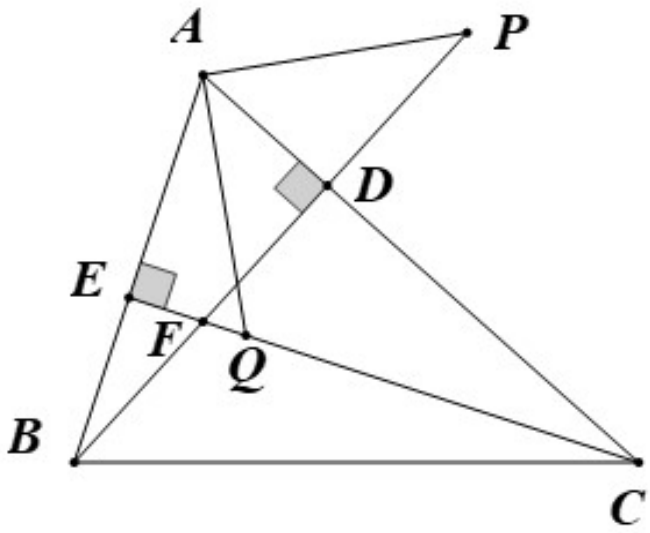
Prove that: BC ⊥ BD and BC = BD.



Key Idea / Approach:

Proof :

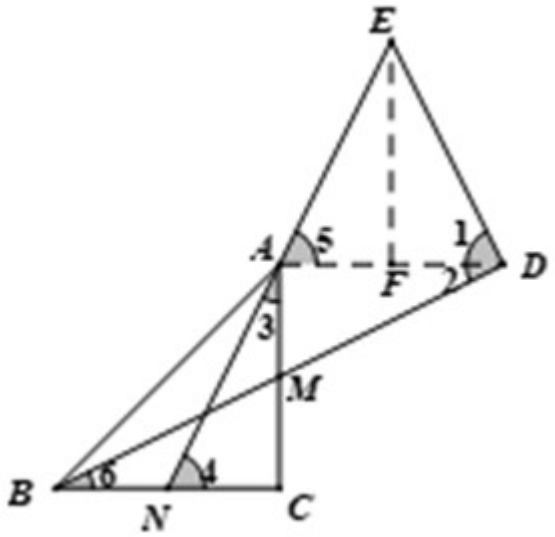
Example 3 *Given: BD and CE are altitudes of ⧋ABC. Point P lies on the extension of BD such that BP = AC. Point Q lies on CE such that CQ = AB. Prove: (1) AP = AQ; (2) AP ⊥ AQ.*



Key Idea / Approach

Proof

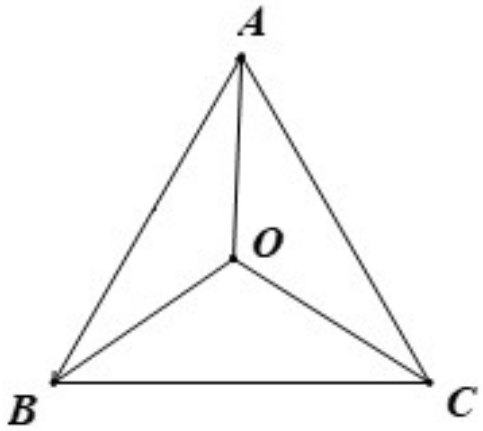
Example 4 *As shown in the figure, $\triangle ABC$ is an isosceles right triangle with $\angle C = 90°$. M and N are the midpoints of sides AC and BC respectively. Point D lies on ray BM such that $BD = 2BM$. Point E lies on ray NA such that $NE = 2NA$. Prove that : $BD \perp DE$.*



Key Idea / Approach

Proof:

Example 5 *In equilateral $\triangle ABC$ with an interior point O, given $\angle AOB = 113°$ and $\angle BOC = 123°$. Consider a triangle whose side lengths are equal to OA, OB, OC. Find the measures of its angles.*

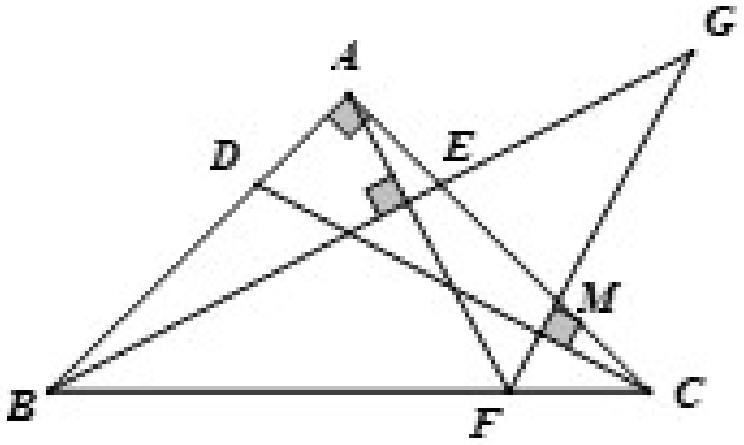


Key Idea / Approach

Proof:

Notes: other rotations

Example 6 *As shown in the figure, in isosceles right $\triangle ABC$ with $\angle BAC = 90°$, $AD = AE$, $AF \perp BE$ intersecting BC at F. From F draw $FG \perp CD$ intersecting the extension of BE at G. Prove that $BG = AF + FG$.*



Key Idea / Approach

Proof

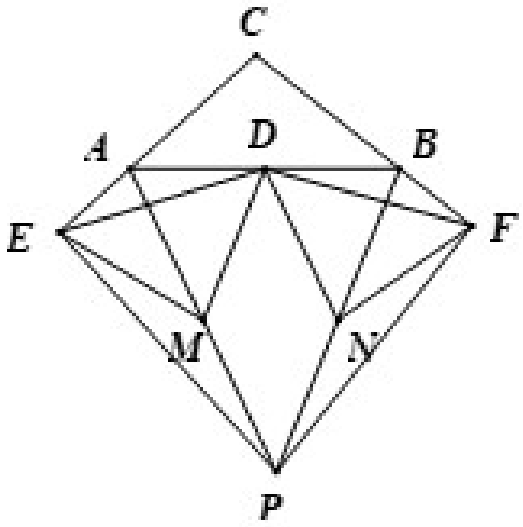
Example 7 *In $\triangle ABC$, $\angle BAC = 5.25°$. AD is the angle bisector of $\angle BAC$. A line through A perpendicular to DA intersects line BC at point M. Given that BM = BA + AC, find the measures of $\angle ABC$ and $\angle ACB$.*

Key Idea / Approach

Calculation:

Notes:

Example 8 *In $\triangle ABC$, D is the midpoint of AB. Extend CA and CB to points E and F respectively such that DE = DF. From E and F, draw lines perpendicular to CA and CB respectively, intersecting at point P. Let M and N be the midpoints of segments PA and PB respectively. Prove: (1) $\triangle DEM \cong \triangle FDN$; (2) $\angle PAE = \angle PBF$.*



Key Idea / Approach

Proof

Summary:

- longer logic processes,
- more frequent ***chasing angles and length around***,
- more diverse geometric transformations,
- more relationships (such as angle bisectors of internal and exterior angle, multiple perpendicular lines, the properties of midpoints and hypotenuse medians, and multiple cases within a single problem).

More importantly…

Homework:

- below this video.
- The answers and hints for the exercises IN the exercises file